SIMULATION OF ENERGY TRANSFER PROCESSES IN LASER SCALPELS

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A mathematical model has been considered for the tips of laser scalpels. Simulation problems have been solved numerically.

In the last few years, laser technology has found ever increasing use in medicine. Such properties of laser radiation as high power density, small beam divergence, and high monochromaticity can be used in surgery, in making laser scalpels. A scalpel is a system that incorporates a pulsed laser, with average radiation power of 40 W and pulse recurrence frequency of 100 Hz, and an optical fiber connected to the focusing system of the laser radiation [1]. Selection of the system is a critical feature in the design of laser scalpels since this system must provide for the necessary beam shape and power density.

In this work, the model for passage of radiation and formation of the thermal field in the optical system (the laser scalpel's tip) is considered. A comparative analysis of different types of tips is given, including ellipsoidal, parabolic, and conical (Fig. 1).

1. Propagation of laser radiation along the scalpel. Laser radiation can be considered with a reasonably high accuracy to be monochromatic with wavelength λ'. Such properties of substances as the reflection coefficient and the coefficient of radiation absorption can be considered constant, independent of the wavelength. When radiation passes through a translucent medium, we can distinguish self-radiation, scattering, and absorption of light. If the wavelength λ' is in the visible range of the spectrum, then at temperatures $T < 1000$ K, the self-radiation can be disregarded. The tips of laser scalpels are usually made of quartz glass and sapphire crystals, the dispersion coefficients of which can be neglected. Therefore, the passage of radiation in the laser scalpel is described by the equation of radiative transfer in an absorbing medium [2]

$$\bar{\Omega}\nabla I(\bar{\Omega}) + \varkappa I(\bar{\Omega}) = 0. \quad (1)$$

In a cylindrical coordinate system, Eq. (1) is of the form

$$\sqrt{1-\gamma^2}\left(\mu\frac{\partial I}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I}{\partial \mu}\right) + \gamma\frac{\partial I}{\partial z} + \varkappa I = 0. \quad (2)$$

Here μ is the cosine of the angle between the projection of the direction $\bar{\Omega}$ on a plane that is perpendicular to the symmetry axis and the radius-vector r, γ is the cosine of the angle between the direction of flight of the photon and the axis of symmetry.

On the lateral surface of the instrument, the phenomena of refraction and partial reflection take place. By assuming that the reflection is mirror reflection, we can write the boundary conditions for Eqs. (1) and (2) in the form

$$I(\bar{\Omega}, \bar{r}) = \delta I(-\bar{\Omega}, \bar{r}) + I^*(\bar{\Omega}, \bar{r}), \ (\bar{\Omega}, \bar{n}) < 0. \quad (3)$$

Here I^* is the intensity of the radiation supplied from the outside.

If the media in contact have real refraction coefficients, then the reflection coefficient $\delta(\bar{\Omega})$ is found from the Fresnel formulas [3]. For media with complex refraction coefficients, the equation to determine $\delta(\bar{\Omega})$ is of the form

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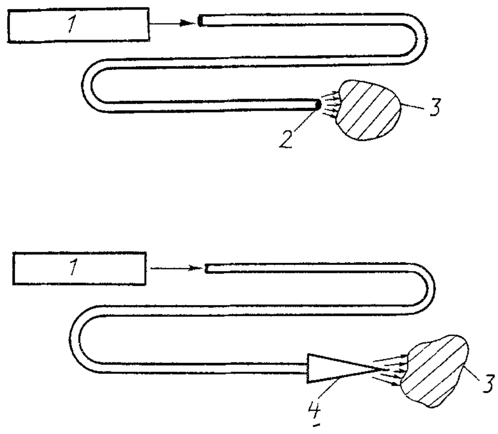

Fig. 1. Arrangement for a laser scalpel: 1) laser; 2) light pipe made of quartz glass, and a tip; 3) epidermis; 4) sapphire focone with light-supplying conductor.

$$\delta = \frac{1}{2}\left[\left|\frac{\bar{n}_2 \cos\beta_1 - \bar{n}_1 \cos\beta_2}{\bar{n}_2 \cos\beta_1 + \bar{n}_1 \cos\beta_2}\right|^2 + \left|\frac{\bar{n}_2 \cos\beta_2 - \bar{n}_1 \cos\beta_1}{\bar{n}_2 \cos\beta_2 + \bar{n}_1 \cos\beta_1}\right|^2\right].$$

Equation (2) with boundary conditions (3) describes the process of laser radiative transfer, and for the case of axial symmetry Eqs. (2) and (3) are four-dimensional with respect to the unknown I. The optimal method for solving such equations is statistical simulation [3, 4]. In this method, the propagation of discrete portions of light ("beams") is tracked. The absorption of the "beams" in the instrument is considered, and, after averaging, the light intensity and the density of radiative energy are determined. The reflection of "beams" from the boundary surface of two media is simulated by methods of linear optics.

The critical point in this method is specification of probability density functions $P(\xi)$ of emission of the "beam" with specified initial parameters ξ. If the probability density function is known, equations for the ray parameters, obtained after transformation of the expression

$$R(\xi) = \int_{-\infty}^{\xi} P(\xi^*)\, d\xi^*,$$

are of the form $\xi = f(R)$, where R is a random number from the interval [0, 1]. When solving the problem numerically, the input energy density is specified in the form of a Gaussian function, the polar angle that characterizes the divergence of the "beam" being given by the equation

$$\theta = \arcsin(\sqrt{R}\sin\theta_{max}).$$

After statistical processing of the results of a numerical experiment, the distribution of densities of light power was obtained in the plane of the exposed tissue.

For conical tips, a diverging radiation beam arises at the output; therefore, conditions are most favorable when the instrument is positioned at a small distance from the epidermis. In this case, the power flux has a maximum value of about $7.5 \cdot 10^5$ W/m^2 (for unit laser radiation power, the radius of the tip's base is equal to 2 mm, and the tip's length equals 2 mm). The distribution pattern of power density for a cone angle of 90 deg is shaped like a Gauss function (Fig. 2).

Ellipsoidal and paraboloidal tips produce a convergent radiation beam, which is focused at a certain distance from the instrument. Their application requires an experimental determination of the optimal distance to tissue. The distribution of power density on epidermis when the scalpel is in contact with tissue does not have a sharply defined maximum in the neighborhood of the symmetry axis, and for ellipsoidal tips, one can observe a decrease in radiation when approaching the symmetry axis (Fig. 2). The maximum power density is obtained

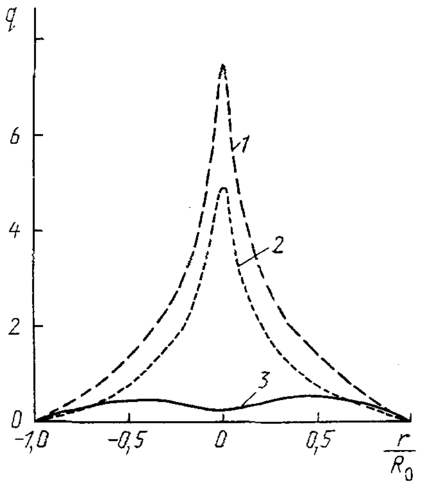

Fig. 2

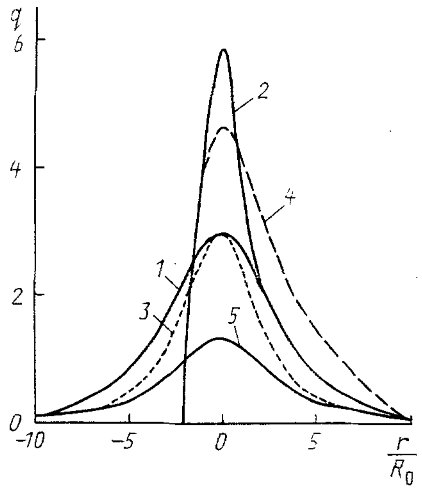

Fig. 3

Fig. 2. Distribution of power density at the output of a tip made of quartz glass ($\cdot 10^5$ W/m^2), with a base radius of 2 mm and ratio between the length and radius of 1/1: 1) conical tip; 2) paraboloidal; 3) ellipsoidal.

Fig. 3. Distribution of power density at the output of the sapphire focone ($\cdot 10^3$ W/m^2) with taper angle ϕ and the angle between the focone axis and a moving coordinate χ: 1) $\phi = 15°$, $\chi = 90°$; 2) 15 and 45; 3) 10 and 90; 4) 10 and 45; 5) 5° and 90°.

as a result of focusing radiation at a certain distance from the point of the instrument. The optimal operational regime depends not only on the tip parameters, but also on the refraction coefficient of the surrounding medium and of the upper layers of epidermis.

Taking account of the thermal regime, tips made of sapphire cones (focones) are optimal. Radiation is supplied to them through a special quartz light conductor whose face plane is polished. The distribution of power density for sapphire focones with taper angles equal to 5, 10, and 15° has a clearly defined maximum in the neighborhood of the point of contact of the laser with epidermis. In Fig. 3, the distribution of power density at the focone's output is shown for two cases: the angle between the symmetry axis of the tip and a dimensionless coordinate is equal to 90° (the scalpel is along the normal) and 45 deg (the scalpel is tilted).

2. Thermal model of laser scalpel. The thermal regime of the tips of the laser scalpels is affected by conductive, convective, and radiative heat exchange. The energy equation describing such processes can be written in the form [5]

$$\operatorname{div}(\lambda \operatorname{grad} T) - \operatorname{div} \overline{W} = 0, \tag{4}$$

where $\overline{W} = \int_0^\infty d\nu \int \overline{\Omega} I_\nu d\Omega$ is the vector of the radiative energy flux.

We analyze the term div $\overline{W}$ that characterizes the contribution of radiation to the total heat exchange. The radiation intensity I_ν entering into (4) can be represented as the superposition of the intensity of laser radiation and of thermal radiation:

$$\overline{W} = \overline{W}_T + \overline{W}_L.$$

For thermal radiation, the mean free path of photons is considerably larger than the characteristic dimensions of the tip; therefore, Planck's approximation is applicable:

$$\operatorname{div} \overline{W}_T = 4\varkappa\sigma T^4. \tag{5}$$

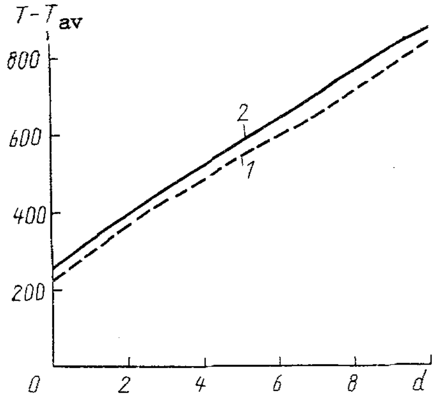

Fig. 4. Dependence of the maximal overheating in the laser scalpel's tip ($T - T_{av}$), K, on the size of the contact zone d, mm, for unit input radiation power and an instrument radius equal to 2 mm: 1) linear problem; 2) nonlinear problem.

For laser radiation, the intensity does not depend on temperature and is determined by the system of Eqs. (2) and (3); therefore, the term div $\overline{W}_L$ can be replaced by its equivalent — the introduction of interior and surface sources into Eq. (4):

$$\operatorname{div} \overline{W}_L = \omega + q\delta(\overline{r} - \overline{r}'),\ \overline{r}' \in \partial G. \tag{6}$$

Equations for determining ω and q are obtained as a result of statistical solution of the transport equation. With the help of the Pomerantsev criterion Po and the Kirpichev criterion Ki, we estimate the contribution of ω and q to heat exchange; Po ≪ 1, so that the contribution of the interior sources is negligible. The main contribution to the heating of the instrument is from the surface sources that arise from contact with nontransparent tissue (Ki ≫ 1).

Equation (4) is solved numerically by finite-difference methods. When simulating the thermal field in the system made up of the laser scalpel and the surrounding medium, it becomes necessary to consider a number of simulation problems and to obtain necessary relationships when analyzing them.

From Eqs. (4) to (6), we can write the simulation problem for the thermal regime of a conical tip in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} + \frac{\omega}{\lambda} = 0,\ (r,\ z) \in G; \tag{7}$$

$$\lambda\frac{\partial T}{\partial n} + \alpha(T - T_{av}) = q,\ (r,\ z) \in \partial G. \tag{8}$$

When numerically solving the given axisymmetrical problem by the method of finite elements, we obtain that the maximum temperature that arises at the point of an instrument made of quartz glass is 450 deg C (the radius of the tip's base is equal to 2 mm, the ratio between the radius and length is 1/2, and the density of the heat flux on a glass—epidermis boundary 2 mm in length is equal to $2 \cdot 10^4$ W/m^2).

Another simulation problem can be represented by the problem of heating the output face of the light conductor, which corresponds to the use of the laser scalpel without a focusing optical system. The given problem with account of (4)-(6) can be written as the system of equations

$$\lambda\frac{\partial^2 T}{\partial z^2} + \frac{2\alpha}{R_0}T + \omega + \frac{2q}{R_0} = 0,\ (r,\ z) \in G; \tag{9}$$

$$\lambda\frac{\partial T}{\partial n} + \alpha(T - T_{av}) = Q,\ (r,\ z) \in \partial G. \tag{10}$$

For numerical solution of system (9) and (10), we use the method of finite differences. We choose a uniform spatial grid $\{z_k\}_{k=1}^{K}$ with the step $h = z_{k+1} - z_k$. Then, for the grid function of temperature, the difference system of equations equivalent to system (9) and (10) assumes the form

$$\frac{\lambda}{h^2}(u_{k+1} - 2u_k + u_{k-1}) - \frac{2\alpha_k}{R_0} u_k + \omega_k + \frac{2q_k}{R_0} = 0, \quad k = \overline{2, K-1}; \tag{11}$$

$$\frac{\lambda}{h}(u_2 - u_1) - \alpha_1 u_1 + \frac{h}{2}\left(\omega_1 + \frac{2q_1}{R_0} - \frac{2\alpha_1}{R_0} u_1\right) = 0; \tag{12}$$

$$\frac{\lambda}{h}(u_{K-1} - u_K) + Q + \frac{h}{2}\left(\omega_K + \frac{2q_K}{R_0} - \frac{2\alpha_K}{R_0} u_K\right) = 0. \tag{13}$$

The given system, obtained by the method of heat balance, is conservative and is of the second order of approximation over the spatial variable. We note that Eqs. (9) and (10) are nonlinear because the coefficients of convective and radiant heat exchange entering into these equations depend on temperature:

$$\alpha = \alpha(T). \tag{14}$$

In order to solve difference equations (11)-(13), one can use an iterative algorithm, on each step of which the thermal field of the scalpel is calculated by the pivotal method, and the coefficient α changes according to new values of the temperature. The indicated algorithm converges, and in order to achieve the desired precision, a few tens of iterations are required. In Fig. 4, the maximum temperature that arises in the light conductor is graphed as a function of the geometrical parameters for the linearized and nonlinearized problem (9) and (10). The temperature of the instrument and the length of the contact zone are related by a practically linear dependence. The heating above the surrounding medium, when the contact zone is equal to 0-10 mm, is 200 to 850 K. The relative error of the linearized problem as compared with the nonlinear one does not exceed 10%.

The results of the numerical calculation of the fields of radiation intensity and temperature distribution can find application in the design and optimization of the tips of laser scalpels. The different types of instruments considered in the article can be used under different conditions, depending on the medical requirements.

NOTATION

I, radiation intensity; T, temperature; T_{av}, average (ambient) temperature; $\bar{r} = (r, z)$, moving coordinates; κ, absorption coefficient; δ, reflection coefficient; β_1 and β_2, angles of incidence and refraction on the boundary of the media with refraction coefficients n_1 and n_2; θ_{max}, maximum beam divergence angle; λ, heat conduction; σ, Stefan-Boltzmann constant; ν, radiation frequency; $Po = \omega L^2/(\lambda T_{av})$, Pomerantsev criterion; $Ki = qL/(\lambda T_{av})$, Kirpichev criterion; α, heat-transfer coefficient; ω, density of interior sources; q, flux density on the lateral surface; Q, flux density on the face plane; R_0, radius of the tip's base; G, region of the tip; ∂G, boundary of the region; $\bar{n}$, outward normal to G; u, grid temperature function.

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